

Mathematics

Advanced GCE

Unit **4730**: Mechanics 3

Mark Scheme for January 2013

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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Annotations and abbreviations

| Annotation in scoris | Meaning |
|----------------------|-------------------------------|
| ✓ and ✕ | |
| BOD | Benefit of doubt |
| FT | Follow through |
| ISW | Ignore subsequent working |
| M0, M1 | Method mark awarded 0, 1 |
| A0, A1 | Accuracy mark awarded 0, 1 |
| B0, B1 | Independent mark awarded 0, 1 |
| SC | Special case |
| ^ | Omission sign |
| MR | Misread |
| Highlighting | |

| Other abbreviations in mark scheme | Meaning |
|------------------------------------|--|
| E1 | Mark for explaining |
| U1 | Mark for correct units |
| G1 | Mark for a correct feature on a graph |
| M1 dep* | Method mark dependent on a previous mark, indicated by * |
| cao | Correct answer only |
| oe | Or equivalent |
| rot | Rounded or truncated |
| soi | Seen or implied |
| www | Without wrong working |

Subject-specific Marking Instructions for GCE Mathematics (OCR) Mechanics strand

- a. Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b. An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme assists in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must be the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the work must be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such methods must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks in the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) consult your Team Leader.

- c. The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood and is not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate to state an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of a mark is specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or evidence than the marking of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a candidate who passes through the correct answer as part of a wrong argument.

- d. When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme indicates otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate is wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, if two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e. The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect work. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) mark is given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will be given for 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is in a different image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f. Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct. It is expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-precision.

When a value is given in the paper

Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to all numerical answers.

When a value is not given in the paper

Accept any answer that agrees with the correct value to 2 s.f.

It should be used so that only one mark is lost for each distinct accuracy error, except for errors due to rounding, which should be penalised only once in the examination. There is no penalty for using a wrong value for g . E marks will be lost except when results agree to the accuracy required.

g. Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examine the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h. For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain the same according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally withheld, but this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, they are established by equivalent working.

'Fresh starts' will not affect an earlier decision about a misread.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i. If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). If the answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting working, in doubt, consult your Team Leader.

j. If in any case the scheme operates with considerable unfairness consult your Team Leader.

| Answer | | Marks | Guidance | | |
|--------|-------|---|---|---|---|
| 1 | | $I^2 = 2.04^2 + 0.9^2 - 2 \times 2.04 \times 0.9 \times \frac{15}{17}$ 1.32 (N) 46.8(°) with initial direction of ball | M1 | | Use of ... missing 2/ for M1 |
| | | | A1 | And attempt to square root | Condone + for - |
| | | | A1 | CAO | (1.3159) |
| | | | M1 | Correct use of sin rule from their diagram oe | Can be in terms of I |
| | | | A1 | CAO | (46.8476) (0.8176 m) Accept 46.7 from us |
| | | [5] | OR $0.9 + I \cos \theta = 0.6 \times 3.4 \times 15/17$ M1 $I \sin \theta = 0.6 \times 3.4 \times 8/17$ M1 square and add to find I^2 ; or divide to find θ M1 I, θ A1 A1 CAO | Allow missing 0.6 a error for these 2 ma | |
| 2 | (i) | Vel unchanged perp to L o C $0.6 \sin 30^\circ = v \cos 30^\circ$ $0.2\sqrt{3}$ (ms ⁻¹) | M1 M1 A1 [3] | | Stated or used Allow 1 sign or trig (0.34641) |
| 2 | (ii) | Use momentum equation $0.3m - 0.6m \cos 30^\circ = am + 0.2\sqrt{3}m \cos 60^\circ$ $(a =) 0.393$ to left | M1 A1ft A1 [3] | Follow through on v Direction must be clearly stated or implied from working. WWW | Allow their v ; allow omission of m m 's not necessary; (0.39282) Away from B/opp d |
| 2 | (iii) | Use of NLR $(0.2\sqrt{3}) \cos 60^\circ - (-0.393) = e(0.6 \cos 30^\circ + 0.3)$ 0.691 | M1 A1ft A1 [3] | Ft on a and v CAO | Allow sign error and (0.69082 or 0.69050) |

| | | Answer | Marks | Guidance | |
|---|------|--|--|---|--|
| 3 | (i) | Use of $F = ma$, using $v \frac{dv}{dx}$ $0.3v \frac{dv}{dx} = 1.5x$ Attempt to rearrange and integrate $v = \sqrt{5x}$ AG | M1* A1 *M1 A1 [4] | $0.3v^2 = 1.5x^2 (+c)$ correct derivation WWW | Allow sign c No need for c . At least one side integrated correctly |
| 3 | (ii) | Integrate to find x in terms of t $\ln x = \sqrt{5}t + c$ $x = e^{\sqrt{5}t}$ $v = \sqrt{5} e^{\sqrt{5}t}$ OR Integrate to find v in terms of t $\frac{dv}{v} = \sqrt{5}dt$ $\ln v = \sqrt{5}t + c$ $\ln v = \sqrt{5}t + \ln(\sqrt{5})$ $v = \sqrt{5} e^{\sqrt{5}t}$ | M1 A1 A1 A1 [4] M1 A1 A1 A1 | $dx/x = \sqrt{5}dt$ and int 1 side correctly CAO Use $\int 0.3 \frac{dv}{v} = 1.5x$ and int 1 side correctly CAO | Need to separate variables No need for c for final answer Must include showing working No need for c for final answer Must include showing working |

| | | Answer | Marks | Guidance | |
|---|------|---|---|---|---|
| 4 | (i) | Conservation of energy $\frac{1}{2}0.4v^2 + \frac{1}{2}0.6v^2 + 0.4g\sin\theta - 0.6g\theta = 0$ $v^2 = 3.92a(3\theta - 2\sin\theta)$ $F = ma$ radially for P $0.4g\sin\theta - R = \frac{0.4v^2}{a}$ $R = -4.704\theta - 7.056\sin\theta$ | M1 M1 A1 M1 A1 M1* A1 *M1 A1 [9] | Attempt to find v^2 dep both earlier M1s AG Manipulation attempted, leading to $a\theta + b\sin\theta$ | Need 4 terms Both KE or b completely correct Allow with sign and No errors Allow sign and trig Allow sign and trig $2.352(-2\theta + 3\sin\theta)$ |
| 4 | (ii) | Using $R = 0$ $(k =) \frac{2}{3}$ | M1 A1 [2] | $0 = -4.704\theta - 7.056\sin\theta$ | Must be from correct |
| 5 | (i) | $2.5g = 36.75 e/3$ $e = 2$ $v^2 = 0^2 + 2g(3 + e)$ $v = 7\sqrt{2}$ $1 \times v = 3.5 V$ Combined speed = $2\sqrt{2}$ (ms ⁻¹) | M1 A1 M1 A1 M1 A1 [6] | P in equilibrium AG | Allow missing g May be implied by Convincing derivation |

| | | Answer | Marks | Guidance |
|---|------|---|--|---|
| 5 | (ii) | change in PE is $3.5gX$ change in KE is $0.5 \times 3.5 (2\sqrt{2})^2$ change in EE is $36.75(X+2)^2/(2 \times 3) - 36.75 \times 2^2/(2 \times 3)$ Use conservation of energy $35X^2 - 56X - 80 = 0$ | B1 B1 M1 A1 M1 A1 [6] | $34.3X$ 14 $\frac{36.75(X+2)^2}{2 \times 3} = \frac{36.75 \times 2^2}{2 \times 3} + 3.5gX \quad \frac{3.5}{2}V^2$ AG Allow sign errors Allow 'x' or 'x + 5' terms or difference Allow sign errors; a term Convincing derivati may see $36.75X^2 - 5$ |
| 6 | (i) | Moments about C for CD $Wl\sqrt{3}/2(\cos 30^\circ) = Ql\sqrt{3}(\cos 30^\circ)$ $(Q =) W/2$ Resolve vert $(R =) \frac{3}{2}W$ | M1 A1 A1 M1 A1 [5] | AG CAO allow M if sin/cos w |
| 6 | (ii) | $X = 0$ Resolve vert for CD or AB $Y = W/2$ Vertically downwards | B1 B1* *B1 [3] | $Y + Q = W$ or $Y + W = R$ |

| Answer | | Marks | Guidance | | |
|--------|-------|--|--|--|--|
| 6 | (iii) | <p>Moments about C for AB</p> <p>$P\cos 30^\circ + F\cos 30^\circ = R\sin 30^\circ$</p> <p>Use P in terms of F</p> <p>Find F in terms of W, or in terms of R</p> <p>$\mu = (F/R) = \sqrt{3}/6$</p> <p>OR Moments about A for AB</p> <p>$W\sin 30^\circ + (Y)\sin 30^\circ + F2\cos 30^\circ = R2\sin 30^\circ$</p> <p>Write Y (and X) in terms of W</p> <p>Find F in terms of W, or in terms of R, or</p> <p>$\mu = (F/R) = \sqrt{3}/6$</p> | <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>[5]</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>M1</p> <p>A1</p> | <p>Correct</p> <p>$F = P$ or other correct 2nd step</p> <p>$F = \frac{\sqrt{3}}{4}W$</p> <p>Accept decimal answers from 0.288675</p> <p>$F = \frac{\sqrt{3}}{4}W$</p> <p>Accept decimal answers from 0.288675</p> | <p>Allow M1 if sign errors; need all terms</p> <p>Allow if missing terms</p> <p>Or getting 'their' F or μR in moment equation</p> <p>Allow M if sin/cos values; need all terms</p> <p>May have X term if</p> |
| 7 | (i) | <p>Use of energy equation</p> <p>$0.5 m (0.3)^2 = mx9.8x0.8x(1 - \cos \theta)$</p> <p>$\theta = 0.107$</p> | <p>M1</p> <p>A1</p> <p>A1</p> <p>[3]</p> | <p>No errors AG</p> | <p>Allow M1 if sign errors; missing and/or missing terms</p> <p>0.107194171</p> |
| 7 | (ii) | <p>Use $F = ma$</p> <p>$\ddot{\theta} = -12.25 \theta$</p> <p>small θ</p> <p>Use of $T = \frac{2\pi}{\omega}$</p> <p>$T = 1.80$</p> | <p>M1</p> <p>A1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>[5]</p> | <p>$m \times 9.8 \sin \theta = -m \times 0.8 \ddot{\theta}$</p> <p>Dep on having seen $\text{acc} = k \sin \theta$ or sight of $\omega = 3.5$</p> | <p>allow M1 if sign errors</p> <p>Allow fraction</p> <p>Rigorous</p> <p>accept $\frac{4\pi}{7}$ (1.795)</p> |

| | | Answer | Marks | Guidance | |
|---|-------|---|--|--|---|
| 7 | (iii) | identifying amplitude as 0.107 Use of $(\theta) = 0.107 \times 3.5 \cos(3.5t)$ Use of $\dot{\theta} = -0.25$ $t = 0.658$ Use of $\theta = 0.107 \sin(3.5t)$ $(\theta =) 0.0797 \text{ rads}$ | B1 M1 A1 A1 M1 A1 [6] | or $\sin(3.5t + \epsilon)$, ϵ not 0 Consistent angle or length ft from velocity equation (matches, ignore sign) accept 5.20° | ft from (i) ft for a and ω (0.6576339) (0.0796678 or 0.07 |

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